

Neutrino interactions with a weak slowly varying electromagnetic field *

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Abstract

We derive the effective action for processes involving two neutrinos and two photons at energies much below the electron mass. We discuss several applications in which one or both photons are replaced by external fields. In particular, Cherenkov radiation and neutrino pair production in weak external fields are investigated for massive Dirac neutrinos.

1 Introduction

The subject of electromagnetic interactions of neutrinos has been widely discussed in the literature, due to their relevance to astrophysics and cosmology. In the standard model, neutrino-photon interactions appear at the one-loop level. To be precise, it is the charged particle running in the loop which, when integrated out, confers its electromagnetic properties to the neutrino. This induces an effective coupling between photons and neutrinos. E.g., a process involving two neutrino legs and one photon (e.g., $\nu \rightarrow \nu\gamma$) which is forbidden in vacuum can become important in the presence of a medium and/or external fields [1, 2]. The standard model processes with two photons, for example, neutrino-photon scattering $\gamma\nu \rightarrow \gamma\nu$, turn out to be highly suppressed in vacuum. In [3] Gell-Mann showed that in the four-Fermi limit of the standard model the amplitude is exactly zero to order G_F ; this is because, according to Yang's theorem [4], which is based on rotational invariance, two photons cannot couple to a $J = 1$ state. Therefore the amplitude is suppressed by the additional factors of ω/m_W , where ω is the photon energy and m_W is the W mass [3, 5, 6, 7]. As a result, the typical vacuum cross sections are exceedingly small (e.g., see [8] for the process $\gamma\nu \rightarrow \gamma\nu$).

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The same reactions in the presence of an external magnetic field are enhanced by the factor $\sim (m_W / m_e)^4 (B/B_c)^2$ for $\omega \ll m_e$ and $B \ll B_c$ as was shown in [9] (extensions to this result can be found in [10, 11]).

In the present letter, we investigate the interaction of two neutrinos with an electromagnetic field to lowest, i.e., second, order in the field by deriving the corresponding effective action. The field can be considered as slowly varying (compared to the Compton wavelength) or, alternatively, as produced by two soft off-shell photons. Since such a field can have any state of angular momentum, the amplitude is generally not suppressed, due to Yang's theorem. A second possibility to circumvent Yang's theorem arises from non-vanishing Dirac neutrino masses, which allow for a $\bar{\nu}\nu$ -pair in a $J = 0$ state.

The calculation as outlined in the following section is actually very simple and, within the four-Fermi limit of the electroweak theory, can solely be based on the famous triangle diagram. The resulting effective action easily reproduces well-known results and finally reveals a variety of interesting new effects that will be briefly discussed in Sec. 3; it includes the production of massive as well as massless neutrinos by varying electromagnetic fields, and Cherenkov radiation for massive neutrinos. We furthermore hint at an enhancement of neutrino oscillations by varying fields without an additional medium.

2 Effective $\nu\nu\gamma\gamma$ Action

For the derivation of the desired effective action, we employ a set of approximations: first, we assume the neutrino energies to be very much smaller than the W - and Z -boson masses, allowing us to use the local limit, i.e., the effective four-Fermi interaction:

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} \bar{\nu} \gamma_\mu (1 + \gamma_5) \nu \bar{E} \gamma^\mu (g_V + g_A \gamma_5) E. \quad (1)$$

Here, E denotes the electron field, $g_V = \frac{1}{2} + 2 \sin^2 \theta_W$ and $g_A = \frac{1}{2}$ for ν_e , and $g_V = -\frac{1}{2} + 2 \sin^2 \theta_W$ and $g_A = -\frac{1}{2}$ for $\nu_{\mu,\tau}$.

Now we place the system into an external electromagnetic field to which the charged fermions can couple, implying the effective action

$$\Gamma_{\text{eff}}[L, A] = \frac{G_F}{\sqrt{2}} \frac{1}{e} \int d^4x L_\mu (g_V \langle j^\mu \rangle^A + g_A \langle j_5^\mu \rangle^A), \quad (2)$$

where we introduced the neutrino current $L_\mu := \bar{\nu} \gamma_\mu (1 + \gamma_5) \nu$ and the electromagnetic currents $j_5^\mu := e \bar{E} \gamma^\mu \gamma_5 E$ and $j^\mu := e \bar{E} \gamma^\mu E$; their expectation values in an external field can be reexpressed in terms of the Green's function G in this field:

$$\langle j_5^\mu \rangle^A = i \text{etr} [\gamma^\mu \gamma_5 G(x, x|A)], \quad (3)$$

and similarly for j^μ . Obviously, the current expectation values correspond to closed fermion loops coupled to the external field. In the following, we omit all contributions from any

charged loop fermion other than electron-positron pairs, since they are suppressed by inverse powers of their mass. Due to the properties of the Dirac algebra (Furry's theorem), only odd numbers of external field couplings contribute to the vector current $\langle j^\mu \rangle^A$, while only even numbers contribute to the axial one $\langle j_5^\mu \rangle^A$.

Since we are furthermore interested in electromagnetic fields whose strength and variation is bound by the scale of the electron mass m , the lowest-order non-trivial contribution to Γ_{eff} in a weak-field expansion arises from the axial current. Expanding the axial current to second order in A^μ leads us to

$$\begin{aligned} \langle j_{5\lambda}(x) \rangle &= \frac{ie}{2} \int \frac{d^4 k_1 d^4 k_2 d^4 k}{(2\pi)^{12}} e^{ix(k_1+k_2)} eA^\mu(k_1) eA^\nu(k_2) \text{tr} \left[g(k_1+k) \gamma_\mu g(k) \gamma_\nu g(k-k_2) \gamma_\lambda \gamma_5 \right] \\ &\quad + \left\{ \mu \leftrightarrow \nu \right\} \\ &=: \frac{e}{2} \int \frac{d^4 k_1 d^4 k_2}{(2\pi)^8} e^{ix(k_1+k_2)} eA^\mu(k_1) eA^\nu(k_2) \Delta_{\mu\nu\lambda}^5(k_1, k_2). \end{aligned} \quad (4)$$

Here we employed the Fourier representations of the free Green's function $g(p)$ as well as the external field $A(p)$. In the last line of Eq. (4), we identified the gauge invariant amplitude of the famous triangle diagram, where k_1 and k_2 are the photons' momenta and k runs around the loop. We may borrow the final result for $\Delta_{\mu\nu\lambda}^5$ from, e.g., [12]. Since we are working in the soft-photon approximation, i.e., $k_1, k_2 \ll m$, the amplitude simplifies considerably and reduces to a sum of terms which are linear in k_1 and quadratic in k_2 or vice versa. After Fourier transforming them back into coordinate space, we obtain the lowest order term of the axial current in a weak-field and soft-momentum expansion¹:

$$\langle j_5^\mu(x) \rangle = \frac{\alpha}{6\pi} \frac{e}{m^2} \left(\partial^\mu \mathcal{G} + (\partial^\alpha F_{\alpha\beta})^* F^{\beta\mu} \right) + \mathcal{O}(1/m^6), \quad (5)$$

where we employed the conventions $*F^{\alpha\beta} = \frac{1}{2}\epsilon^{\alpha\beta\kappa\lambda} F_{\kappa\lambda}$, and $\mathcal{G} := -\frac{1}{4}F^{\mu\nu}*F_{\mu\nu}$. Upon insertion into Eq. (2), the effective action for the lowest-order neutrino-photon interaction reads:

$$\Gamma_{\text{eff}}[L, A] = \frac{G_F g_A}{\sqrt{2}} \frac{\alpha}{6\pi} \frac{1}{m^2} \int d^4 x \left(-(\partial^\mu L_\mu) \mathcal{G} + (\partial^\alpha F_{\alpha\beta}) (L_\mu *F^{\beta\mu}) \right) + \mathcal{O}(1/m^4). \quad (6)$$

This equation represents the central result of our investigation. Now, if all external particles are on shell, we get $\partial_\mu L^\mu = 0$ for massless neutrinos, and $\partial_\alpha F^{\alpha\beta} = 0$ for free photons. Therefore, the amplitude vanishes, which is nothing but the manifestation of Yang's theorem. However, interesting effects can be discovered for deviations from this (standard model) on-shell behavior. A first glance at this will be outlined in the next section.

¹In principle, the axial current in Eq. (3) is an ill-defined operator identity because of the presence of the anomaly; however, in the here-considered "large electron mass" expansion, the anomaly is not present, since it is mass independent. In other words, the anomaly does not contribute to the neutrino-photon interaction as studied in this work.

Let us finally mention that Eq. (6) represents the second-order analogue of the effective action derived by Dicus and Repko [13], which describes neutrino interactions with three external photon lines; the latter characterizes the lowest-order effective theory for (standard model) on-shell interactions.

3 Applications

By construction, Eq. (6) represents the effective action for two-photon two-neutrino processes, and we can easily derive the matrix element for, e.g., the process $\bar{\nu}\nu \rightarrow \gamma\gamma$ for massive neutrinos:

$$\mathcal{M}(\bar{\nu}_l \nu_l \rightarrow \gamma(k, \epsilon), \gamma(k', \epsilon')) = -\frac{G_F(2g_A)}{\sqrt{2}} \frac{\alpha}{6\pi} \frac{m_\nu}{m^2} \bar{v}_l^\nu \gamma_5 u_l^\nu \epsilon^{\mu\nu\alpha\beta} k_\mu k'_\nu \epsilon_\alpha \epsilon'_\beta, \quad (7)$$

which is in perfect agreement with [12] for $g_A = \frac{1}{2}$ for ν_e and $g_A = -\frac{1}{2}$ for $\nu_{\mu,\tau}$, as it should be. We observe that Eq. (7) arises from the first term of Eq. (6) only, since the second term vanishes for on-shell photons.

3.1 Cherenkov radiation by massive neutrinos in magnetic fields

A review of this effect for massless neutrinos has been performed in study [14]. The emission of Cherenkov radiation by neutrinos propagating perpendicular to a magnetic field becomes possible, because the phase velocity of soft photons in a magnetic field is smaller than in vacuum $k^2 \sim -\alpha \frac{B^2}{B_{\text{cr}}^2}$, $B_{\text{cr}} = \frac{m_e^2}{e}$ (for a review, see [15]). It can be shown that the contributions of the vector part of Γ_{eff} in Eq. (2) to the transition rate are proportional to k^2 and hence are suppressed by additional orders of α^2 . The same holds for the axial part of Eq. (2) if one considers \perp -photons which are perpendicularly polarized compared to the field direction. Hence, only parallelly polarized \parallel -photons can be emitted as Cherenkov radiation.

Since $\partial^\alpha F_{\alpha\beta} \sim k^2 \epsilon_\beta$, the contributions of the second-order effective action (6), can similarly be neglected for massless neutrinos, so that the lowest order contribution to the well-known transition rate arises from the terms $\sim 1/m^6$ in Γ_{eff} corresponding to a pentagon diagram. The situation changes for massive neutrinos when the first term of Eq. (6) no longer vanishes.

Associating one field strength tensor with the external magnetic field and the other with the emitted \parallel -photon of frequency ω , the matrix element reads:

$$\mathcal{M}(\nu(p) \rightarrow \nu(p'), \gamma(k)) = i \frac{G_F g_A}{\sqrt{2}} \frac{\alpha}{3\pi} m_\nu \omega \frac{B}{m^2} \bar{u}_l^\nu(p') \gamma_5 u_l^\nu(p). \quad (8)$$

For neutrino energies below the e^+e^- pair production threshold, and neglecting the small

deviations from the photon light cone, the transition rate yields:

$$\begin{aligned}
\Gamma_{\nu \rightarrow \nu \gamma} &= \frac{1}{2E} \sum_{\text{pol.}} \int \frac{d^3 k}{(2\pi)^3 2\omega} \int \frac{d^3 p'}{(2\pi)^3 2E'} (2\pi)^4 \delta^4(p - (p' + k)) |\mathcal{M}|^2 \\
&= \frac{7}{2^9 \cdot 3^4 \cdot 5^2} \frac{\alpha^2}{\pi^4} m (G_F m^2)^2 \left(\frac{E}{m}\right)^3 \left(1 - \frac{E_{\min}^2}{E^2}\right)^5 \left(\frac{m_\nu}{m}\right)^2 \left(\frac{B}{B_{\text{cr}}}\right)^4 \theta(E - E_{\min}) \\
&= 2.6 \cdot 10^{-14} \text{ s}^{-1} \left(\frac{E}{m}\right)^3 \left(1 - \frac{E_{\min}^2}{E^2}\right)^5 \left(\frac{m_\nu}{m}\right)^2 \left(\frac{B}{B_{\text{cr}}}\right)^4 \theta(E - E_{\min}) \quad (9)
\end{aligned}$$

where $B_{\text{cr}} = \frac{m^2}{e}$ and

$$E_{\min} := \sqrt{\frac{45\pi}{7\alpha}} m_\nu \frac{B_{\text{cr}}}{B}. \quad (10)$$

The existence of such a mass-dependent threshold energy E_{\min} for the incoming neutrino arises from the Cherenkov condition: the neutrino must move “faster than light” in the B -field background. It is easy to check that the transition rate (9) never wins out over the mass-independent contribution as cited in [14], which is proportional to $(E/m)^5 (B/B_{\text{cr}})^6$. This statement holds for any value of the neutrino mass in the low-energy domain.

We would like to stress that this zero-result is a non-trivial statement and should serve as a counterexample for the general belief that non-zero neutrino masses always open a window in parameter space for new effects.

3.2 $\bar{\nu}\nu$ -pair emission by varying electromagnetic fields

Although electron-positron pair emission by varying electromagnetic fields belongs to standard textbook knowledge (see, e.g., [16]), it is far from being phenomenologically important, due to the enormous threshold frequency $\omega_{\text{cr}} = 2m = 1.5 \cdot 10^{21} \text{ Hz}$ with which the field must oscillate. The effective action Eq. (6) reveals a similar mechanism for neutrinos which benefits from the smallness of the neutrino mass.

Let us consider spacetime regions where a varying electromagnetic field satisfies the vacuum Maxwell equations; then only the first term of Eq. (6) contributes to the pair production matrix element:

$$\mathcal{M}(\mathcal{G}(k) \rightarrow \bar{\nu}(p'), \nu(p)) = i \frac{G_F g_A}{\sqrt{2}} \frac{\alpha}{3\pi} \frac{m_\nu}{m^2} \mathcal{G}(k) \bar{v}_l^{\bar{\nu}}(p') \gamma_5 u_l^\nu(p), \quad k = p + p', \quad (11)$$

where $\mathcal{G}(k)$ denotes the Fourier transform of $\mathcal{G}(x)$. The production probability is given by

$$\begin{aligned}
W &= \int d^4 k |\mathcal{G}(k)|^2 \int \frac{d^3 p}{(2\pi)^3 2E} \int \frac{d^3 p'}{(2\pi)^3 2E'} \delta^4(k - (p + p')) \sum_{\text{pol.}} |\mathcal{M}|^2 \\
&= \frac{G_F^2 \alpha^2}{36 (2\pi)^7} \frac{m_\nu^2}{m^4} \int d^4 k |\mathcal{G}(k)|^2 k^2 \left(1 - \frac{4m_\nu^2}{k^2}\right)^{1/2} \theta\left(1 - \frac{4m_\nu^2}{k^2}\right). \quad (12)
\end{aligned}$$

In order to obtain an illustrative estimate of the order of magnitude of this effect, let us simply take $\mathcal{G}(k) = \mathbf{E}_0 \cdot \mathbf{B}_0 (2\pi)^4 \delta^3(\mathbf{k}) \delta(\omega - 2\omega_0)$. For this field configuration, we obtain the production probability per volume and time

$$\frac{W}{VT} \simeq 5.11 \left(\frac{m_\nu}{1\text{eV}} \right)^2 \left(\frac{\omega_0}{1\text{eV}} \right)^2 \frac{(\mathbf{E}_0 \cdot \mathbf{B}_0)^2}{B_{\text{cr}}^4} \left(1 - \frac{m_\nu^2}{\omega_0^2} \right)^{1/2} \theta(\omega_0 - m_\nu) \text{cm}^3\text{s}, \quad (13)$$

in units of cm^3 and seconds. Obviously, the threshold frequency is equal to the neutrino mass, e.g., in the strong ultraviolet for neutrino masses at the eV-scale. Equation (13) can also be interpreted as the number of pairs produced in the system and volume under consideration [17].

It is instructive to compare this pair-production probability with the one for e^+e^- -pairs: $W_{e^+e^-} \sim \int (E^2 - B^2)$. Each process is triggered by a different invariant of the electromagnetic field revealing its vector or axial vector character.

Another neutrino pair-creation mechanism was proposed by Kachelriess [18], which is caused by a density gradient of background fermions (e.g., neutrons). It is interesting to observe that the two mechanisms exhibit very different sensitivities to the neutrino mass: while the above-studied mechanism obeys a power law, the latter decays exponentially with increasing m_ν , since this effect is based on the Schwinger mechanism.

4 Comments

In the present letter, we derived the low-energy effective action for processes involving two neutrinos and two photons (or couplings to an external field) and discussed several immediate applications.

The here-considered effects arise essentially from the first term of the effective action in Eq. (6). Of course, we could have also studied field configurations inducing similar effects that are triggered solely by the second term of Eq. (6), e.g., purely magnetic field configurations with spatial variation. In this case, we should, however, confine the investigations to situations where the fields are not produced by standard model currents, since those currents will also couple directly and without $\mathcal{O}(\alpha)$ -suppression to the neutrino current. Such fields might be produced by, e.g., aligned spins, as is possible in neutron stars, or variable Higgs condensates which might be responsible for primordial magnetic fields. Upon insertion of these field configurations into the above-given calculations, the neutrino mass terms get effectively replaced by spatial derivatives on the magnetic field. Hence, in order to obtain effects of comparable amount, the characteristic length scale L over which the magnetic field varies must be of the order of the Compton wavelength of the neutrino $L \sim 1/m_\nu = 1.9 \cdot 10^{-5} \text{cm} (1\text{eV}/m_\nu)$. Therefore, we expect the above-presented examples to be of greater importance.

Except for mass differences, the above-discussed examples are not directly sensitive to different neutrino flavors, since $g_A^2 = 1/4$ holds for all flavors ($g_A = 1/2$ for electron neutrinos and $g_A = -1/2$ for others). A different example can be inferred from the ax-

ial current Eq. (5): consider a spatially constant electromagnetic vacuum field varying in time with non-vanishing $\mathbf{E} \cdot \mathbf{B} \neq 0$. Its contribution to the axial charge density is $\frac{g_A}{e} \langle j_5^0 \rangle = \frac{\alpha}{6\pi} \frac{g_A}{m^2} \frac{d}{dt} \mathbf{E} \cdot \mathbf{B}$. Therefore, such a field configuration is in complete analogy to a polarized medium. In this way, a neutrino propagating in such a field can be subject to an enhancement of flavor oscillations, similarly to a propagation in matter.

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